Online Appendix for
Missing Growth from Creative Destruction

Philippe Aghion, Antonin Bergeaud

Timo Boppart, Peter J. Klenow, Huiyu Li

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A Imputation in the CPI

For compiling a price index, accurately adjusting for quality changes poses a challenge. Let v denote an item produced at date t and which is replaced by a new item $v+1$ at date $t+1$. To integrate the corresponding item change in the overall price index, the statistical office needs to infer a value for either price $P(v+1,t)$ or price $P(v,t+1)$ when it has information only about $P(v,t)$ and $P(v+1,t+1)$. According to the U.S. General Accounting Office (1999) and to the Handbook of Methods from U.S. Bureau of Labor Statistics (2015), the BLS largely chooses among four possible courses of action to handle these *item substitutions*.[1]

The first course of action simply involves setting

$$P(v+1,t) = P(v,t).$$

This no-adjustment strategy is pursued by the BLS when it deems the new and old item as *comparable*, by which the BLS means that the old and new items are essentially the same, so that no quality difference exists between the two items.

The interesting case is when the BLS judges the new and old items to be *noncomparable*. Then, the BLS typically chooses between three remaining strategies. First is *direct quality adjustment*. This is when the BLS can perform hedonic regressions or has information on manufacturers' production costs. Direct quality adjustment involves the BLS setting

$$P(v+1,t) = P(v,t) \cdot QA(t).$$

Viewed through the lens of our model, BLS quality adjustments are an estimate of the step size of innovations.

For those noncomparable substitutions where the BLS lacks the

[1] We use italics to highlight terminology used by the BLS.

information to make direct quality adjustments, it resorts to *class-mean imputation* or *linking*. Class-mean imputation is based on the rate of price changes experienced by other item substitutions — those which the BLS considers comparable or can directly adjust. Linking, meanwhile, uses the average rate of price change among items without substitution, items with comparable substitutions, and items with noncomparable substitutions subject to direct quality adjustments. Both imputations are usually carried out within the item's category or category-region.

Based on Klenow and Kryvtsov (2008), the BLS judged 52% of item substitutions to be comparable from 1988–2004; the prices for these items entered the CPI without adjustment. The remaining 48% (the noncomparable substitutions) broke down as follows:[2]

- 31.4% direct quality adjustments
- 32.4% class-mean imputations
- 36.2% linking.

To estimate the fraction of creative destruction innovations that were effectively subject to imputation based on all surviving items (those not creatively destroyed), we make the following three assumptions:

1. Comparable item substitutions do not involve any innovation.
2. Direct adjustments are implemented when incumbents improve their own products (OI).
3. Creative destruction (CD) results in imputation by class-mean or linking in the proportions stated above.

[2]These figures are quite close to those in the publicly available statistics for 1997 in U.S. General Accounting Office (1999).

Under these assumptions, we estimate that creative destruction (CD) innovations were treated with the equivalent of all-surviving-items imputation 90% of the time from 1988–2004. To see why, let D, C, and L denote the numbers of item substitutions subject to direct adjustment, class-mean imputation, and linking, respectively. Let N denote the number of comparable item substitutions.

The number of item substitutions for which some form of imputation is done is $L + C$. The imputation in the two strategies, however, is based on different sets of products. Whereas linking imputes from all surviving products (as in our theoretical model), class-mean imputation is based on other (comparable and noncomparable) substitutions. We are looking for the fraction E of the products $L + C$ for which imputation is effectively based on all surviving products, as opposed to just those surviving products with incumbent own innovations (fraction $1 - E$). These include all cases of linking plus a fraction (call it x) of class-mean imputations:

$$E = \frac{L + x \cdot C}{L + C}. \tag{1}$$

How do we determine x? Class-mean imputations C use a weighted average for inflation from item substitutions for which there was either no adjustment (fraction $N/(D + N)$) or a direct adjustment (fraction $D/(D + N)$). Since 48% of all substitutions over the period 1988–2004 were noncomparable (31.4% of which were direct adjustments) and 52% of all substitutions were comparable, we get:

$$\frac{D}{D + N} = \frac{0.314 \cdot 0.48}{0.314 \cdot 0.48 + 0.52} \approx 0.225.$$

Using the assumptions above and results from Klenow and Kryvtsov (2008), the fraction of incumbent own-innovations (OI) among surviving products (those not creatively destroyed) is $\lambda_i \approx 0.60\%$ monthly.[3] If the fraction of direct

[3] Together with a monthly rate of product exit of 3.9% this number is obtained as $(0.039 \cdot 0.48 \cdot 0.314)/(0.961 + 0.039[0.52 + 0.48 \cdot 0.314])$.

quality adjustments in class-mean imputations was also 0.60%, we would say class-mean imputation is just like linking (imputation based on all products not creatively destroyed). Because the fraction of direct quality adjustments in class-mean imputations (at 22.5%) was higher than 0.60%, we infer that class-mean imputation puts extra weight on OI:

$$\frac{D}{D+N} = x \cdot \lambda_i + (1-x) \cdot 1, \tag{2}$$

where x is the weight on all surviving items (only fraction λ_i of which were innovations) and $1 - x$ is the weight on those surviving products which did experience incumbent innovations. Rearranging (2) and using the above percentages we get

$$x = \frac{N/(D+N)}{1-\lambda_i} \approx \frac{0.775}{1-0.0060} \approx 0.780.$$

Thus, class-mean imputation effectively puts 78% weight on all surviving items and 22% weight on innovating survivors. Given that class-mean imputation was used 32% of time time and linking was used 36% of time, we estimate that the BLS used imputation based on all surviving items the equivalent of 90% of the time from 1988–2004. More exactly, we substitute the numerical values for x, L and C into (1) to get

$$E = \frac{L + x \cdot C}{L + C} \approx \frac{0.362 \cdot 1 + 0.324 \cdot 0.780}{0.362 + 0.324} \approx 0.896.$$

B Proofs

B.1 Proof of Proposition 1

Proof. The first-order conditions when maximizing (1) subject to the (budget) constraint $M = \int_0^N y(j)p(j)dj$, can be written as

$$\xi\, p(j) = q(j)^{\frac{\sigma-1}{\sigma}} y(j)^{-\frac{1}{\sigma}} \left(\int_0^N \left[q(j')y(j')\right]^{\frac{\sigma-1}{\sigma}} d(j') \right)^{\frac{1}{\sigma-1}}, \ \forall j \in [0, N],$$

where ξ is the Lagrange multiplier attached to the budget constraint. Integrating both sides of this equation over all j's and combining it with (1) yields

$$\xi = \frac{Y}{M} = \frac{1}{P}.$$

Together with the above first-order conditions, this yields (4). Next, to derive expression (5) for P, note that (4) implies that

$$p(j)y(j) = \frac{M}{P} q(j)^{\sigma-1} P^{\sigma} p(j)^{1-\sigma}.$$

Integrating both side of this equation over all j's then immediately yields (5). Finally, substituting for the equilibrium $p(j)$ using (2) in (5) yields equation (6). This establishes the proposition. ■

B.2 Proof of Proposition 2

Proof. Taking gross growth factors of both sides of (6) gives

$$\frac{P_{t+1}}{P_t} = \frac{W_{t+1}}{W_t} \left(\int_0^{N_t} q_t(j')^{\sigma-1} dj' \right)^{\frac{1}{\sigma-1}} \left(\int_0^{N_{t+1}} q_{t+1}(j)^{\sigma-1} dj \right)^{\frac{1}{1-\sigma}}. \qquad (3)$$

Next, note that the term, $\int_0^{N_{t+1}} q_{t+1}(j)^{\sigma-1}dj$, can be written as

$$\int_0^{N_{t+1}} q_{t+1}(j)^{\sigma-1}dj = \int_0^{N_t} q_{t+1}(j)^{\sigma-1}dj + \int_{N_t}^{N_{t+1}} q_{t+1}(\iota)^{\sigma-1}d\iota. \tag{4}$$

Furthermore, with Assumption 1 and $\frac{N_{t+1}-N_t}{N_t} = \lambda_n$, we obtain

$$\int_{N_t}^{N_{t+1}} q_{t+1}(\iota)^{\sigma-1}d\iota = \lambda_n \gamma_n^{\sigma-1} \int_0^{N_t} q_t(j)^{\sigma-1}dj. \tag{5}$$

The first term on the right-hand side of (4), $\displaystyle\int_0^{N_t} q_{t+1}(j)^{\sigma-1}dj$, can be rewritten as

$$\begin{aligned} \int_0^{N_t} q_{t+1}(j)^{\sigma-1}dj &= \gamma_d^{\sigma-1} \int_{\iota\in\mathcal{D}_t} q_t(\iota)^{\sigma-1}d\iota \\ &+\gamma_i^{\sigma-1} \int_{j'\in\mathcal{O}_t} q_t(j')^{\sigma-1}dj' \\ &+ \int_{\iota'\in\tilde{\mathcal{N}}_t} q_t(\iota')^{\sigma-1}d\iota'. \end{aligned} \tag{6}$$

where $\mathcal{D}_t$ and $\mathcal{O}_t$ is the set of products with a successful creative destruction or incumbent own innovation and $\tilde{\mathcal{N}}_t = [0, N_t] \setminus \{\mathcal{D}_t \cup \mathcal{O}_t\}$ is the set of surviving incumbents that do not improve the quality of their product between t and $t+1$. We also know that $|\mathcal{D}_t| = \lambda_d N_t$ and $|\mathcal{O}_t| = (1-\lambda_d)\lambda_i N_t$. Then, because the arrival rate of an innovation is independent of $q_t(j)$ (and there is a continuum of varieties) the distribution of productivity of the varieties with and without innovation coincide and then by the law of large numbers we have

$$\begin{aligned} \int_{\iota\in\mathcal{D}_t} q_t(\iota)^{\sigma-1}d\iota &= \lambda_d \int_0^{N_t} q_t(j)^{\sigma-1}dj, \\ \int_{j'\in\mathcal{O}_t} q_t(j')^{\sigma-1}dj' &= (1-\lambda_d)\lambda_i \int_0^{N_t} q_t(j)^{\sigma-1}dj, \\ \int_{\iota'\in\tilde{\mathcal{N}}_t} q_t(\iota')^{\sigma-1}d\iota' &= [1-\lambda_d-(1-\lambda_d)\lambda_i] \int_0^{N_t} q_t(j)^{\sigma-1}dj. \end{aligned}$$

This in turn implies that (6) can be expressed as

$$\frac{\int_0^{N_t} q_{t+1}(j)^{\sigma-1}dj}{\int_0^{N_t} q_t(j)^{\sigma-1}dj} = 1 + \lambda_d\left(\gamma_d^{\sigma-1} - 1\right) + (1-\lambda_d)\lambda_i\left(\gamma_i^{\sigma-1} - 1\right). \tag{7}$$

Putting equations (3), (5), and (7) together establishes the proposition. ■

B.3 Proof of Proposition 3

Proof. Under Assumption 2 we have

$$\left(\widehat{\frac{P_{t+1}}{P_t}}\right) = \frac{W_{t+1}}{W_t}\left(\int_{\mathcal{I}_t} q_t(j')^{\sigma-1}dj'\right)^{\frac{1}{\sigma-1}}\left(\int_{\mathcal{I}_t} q_{t+1}(j)^{\sigma-1}dj\right)^{\frac{1}{1-\sigma}}, \tag{8}$$

where $\mathcal{I}_t = [0, N_t] \setminus \mathcal{D}_t$ is the set of surviving products with the same producer in period t and $t+1$. Note that a fraction λ_i of these surviving products experiences incumbent own innovation (and the quality improves by a factor of γ_i) whereas for the remaining fraction, $1-\lambda_i$, quality remains unchanged. Hence, we have $\int_{\mathcal{I}_t} q_{t+1}(j)^{\sigma-1}dj = \left(\int_{\mathcal{I}_t} q_t(j')^{\sigma-1}dj'\right)\left[1-\lambda_i+\lambda_i\gamma_i^{\sigma-1}\right]$. Using this equation in (8) and replacing γ_i and λ_i by their estimates yields (9). ■

B.4 Proof of Proposition 5

Proof. Using the price setting behavior of the firms, (2), yields for the market share growth $\frac{S_{I_t,t+1}}{S_{I_t,t}} = \left(\frac{W_{t+1}/P_{t+1}}{W_t/P_t}\right)^{1-\sigma}\frac{\int_{\mathcal{I}_t} q_{t+1}(j')^{\sigma-1}dj'}{\int_{\mathcal{I}_t} q_t(j)^{\sigma-1}dj}$. Now note that a fraction λ_i of continuers experience incumbent own innovation whereas for the remaining fraction, $1-\lambda_i$, quality remains unchanged. Hence, we have $\int_{\mathcal{I}_t} q_{t+1}(j')^{\sigma-1}dj' = \int_{\mathcal{I}_t} q_t(j)^{\sigma-1}dj\left[1-\lambda_i+\lambda_i\gamma_i^{\sigma-1}\right]$, which establishes the proposition. ■

C An illustrative example: the Cobb-Douglas case

Even though this may not be the most realistic case, we use the special case where the production technology for the final good is Cobb-Douglas to illustrate how creative destruction can lead to missing growth. Hence, let us consider the limit case where final output is produced according to the Cobb-Douglas technology

$$Y = N \exp\left[\frac{1}{N}\int_0^N \log\left[q(j)y(j)\right] dj\right]. \tag{9}$$

We assume the number of varieties N is fixed here because there is no love-of-variety under Cobb-Douglas aggregation.

Aggregate price index Since the final goods sector is competitive, demand for product $y(j)$ is

$$y(j) = \frac{PY}{Np(j)},$$

where $p(j)$ is the price of intermediate good j. P is the price index:

$$P = \exp\left(\frac{1}{N}\int_0^N \log\left[p(j)/q(j)\right] dj\right).$$

Under the optimal price setting rule we get

$$P = \mu W \exp\left(-\frac{1}{N}\int_0^N \log\left(q(j)\right) dj\right).$$

The true inflation rate can then be expressed as

$$\frac{P_{t+1}}{P_t} = \frac{W_{t+1}}{W_t}\gamma_i^{-(1-\lambda_d)\lambda_i}\gamma_d^{-\lambda_d}.$$

Measured inflation and missing growth Under Assumption 2 measured inflation becomes

$$\widehat{\left(\frac{P_{t+1}}{P_t}\right)} = \frac{W_{t+1}}{W_t}\gamma_i^{-\lambda_i}.$$

Consequently, we obtain for missing growth

$$MG = \lambda_d \cdot (\log \gamma_d - \lambda_i \log \gamma_i). \tag{10}$$

This missing growth from creative destruction can be decomposed as

$$\lambda_d (\log \gamma_d - \lambda_i \log \gamma_i) = \lambda_d (1 - \lambda_i) \log \gamma_i + \lambda_d (\log \gamma_d - \log \gamma_i) .$$

The first term in this decomposition captures the fact that not all incumbents innovate, whereas the second term captures the step size differential between creative destruction and incumbent own innovation.

Numerical example The Cobb-Douglas case with the following calibration replicates the motivating example of the introduction. Let us assume: (i) no variety expansion; (ii) the same step size for incumbent own innovation (OI) and for creative destruction (CD), i.e., $\gamma_i = \gamma_d = \gamma$, and (iii) annualized arrival rates λ_i and λ_d of OI and CD by new entrants that are both equal to 10%. Finally, assume that the common step size is $\gamma_i = 1.1$, or 10%. Then *measured* annual real output growth is equal to 1.1% ($\lambda_i \log \gamma_i = .011$). From (10), the annual rate of missing growth from creative destruction is equal to

$$MG = 10\% \cdot (1 - 10\%) \cdot 10\% = 0.9\%.$$

True growth is 2% in this example. Hence, roughly half of the growth is missed due to imputation. Although this is just an illustrative exercise, we will see in the next sections that this simple example is not far off from what we obtain using firm-level data on employment dynamics to infer the step sizes and frequencies of each type of innovations.

D Heterogeneous elasticities and varying markups

In this section of the Online Appendix, we discuss how our analysis of missing growth can be extended: (i) to the case of non-CES production technologies; and (ii) to accommodate varying markups.

D.1 Non-CES production elasticities

Let us first recall that the main equation used in the market share approach in our core analysis makes use of the CES production technology for the final good (i.e., of the assumption of a uniform elasticity of substitution σ across intermediate inputs). There we related the market share of product j to its quality adjusted price relative to the price index, according to the equilibrium expression:

$$s_t(j) \equiv \frac{p_t(j)x_t(j)}{M_t} = \left(\frac{P_t}{p_t(j)/q_t(j)}\right)^{\sigma-1}, \tag{11}$$

where P_t is the "true" price index, M_t are nominal expenditure, $p_t(j)/q_t(j)$ is the quality-adjusted price, and σ is the constant elasticity of substitution. From this it is clear that the choice of the value of σ is quantitatively important and so is also the assumption that this elasticity is constant.

Now consider the case where the technology for producing the final good is general constant return to scale production function, with real output Y_t given by

$$Y_t = \frac{M_t}{P\left(p_t(1), ..., p_t(N_t)\right)}, \tag{12}$$

where $P\left(p_t(1), ..., p_t(N_t)\right)$ is the true price index.

Roy's identity yields the Marshallian demand

$$x_t(j) = \frac{P_j\left(p_t(1), ..., p_t(N_t)\right)}{P\left(p_t(1), ..., p_t(N_t)\right)} M_t, \tag{13}$$

where $P_j\left(p_t(1), ..., p_t(N_t)\right) \equiv \frac{\partial P(p_t(1),...,p_t(N_t))}{\partial p_t(j)}$.

In this case the share spent on product j is given by

$$s_j(t) \equiv \frac{p_t(j)x_t(j)}{M_t} = \frac{P_j\left(p_t(1), ..., p_t(N_t)\right)p_t(j)}{P\left(p_t(1), ..., p_t(N_t)\right)}, \tag{14}$$

and the elasticity of that share with respect to the firm's own price is given by

$$\frac{\partial s_t(j)}{\partial p_t(j)}\frac{p_t(j)}{s_t(j)} = \frac{\partial \frac{P_j(p_t(1),...,p_t(N_t))}{P(p_t(1),...,p_t(N_t))}}{\partial p_t(j)} \frac{p_t(j)}{\frac{P_j(p_t(1),...,p_t(N_t))}{P(p_t(1),...,p_t(N_t))}} + 1. \tag{15}$$

Thus, if we denote the (local) price elasticity of demand as ,

$$-\sigma_j\left(p_t(1), ..., p_t(N_t)\right) = \frac{\partial \frac{P_j(p_t(1),...,p_t(N_t))}{P(p_t(1),...,p_t(N_t))}}{\partial p_t(j)} \frac{p_t(j)}{\frac{P_j(p_t(1),...,p_t(N_t))}{P(p_t(1),...,p_t(N_t))}},$$

the market share of intermediate producer j is approximated by a similar expression to (11), namely:

$$s_j(t) = \left(\frac{P_t}{p_t(j)}\right)^{\sigma_j(\cdot)-1}, \tag{16}$$

where $\sigma_j(\cdot)$ is the *local* elasticity.

Hence, as long as we know the *local* elasticity $\sigma_j(\cdot)$ the "market share approach" can still be used to quantify missing growth.

Suppose the elasticity of substitution differs between different type of inputs. Which elasticity of substitution should then be used in the market share approach? More specifically, suppose we have the following production technology for the final good:

$$Y^{\frac{\sigma_B-1}{\sigma_B}} = \left[\int_{\mathcal{I}} \left[q(j)y(j)\right]^{\frac{\sigma_I-1}{\sigma_I}} dj\right]^{\frac{\sigma_I}{\sigma_I-1}\frac{\sigma_B-1}{\sigma_B}} + \left[\int_{\mathcal{N}\setminus\mathcal{I}} \left[q(j)y(j)\right]^{\frac{\sigma_N-1}{\sigma_N}} dj\right]^{\frac{\sigma_N}{\sigma_N-1}\frac{\sigma_B-1}{\sigma_B}},$$

where $\mathcal{I}$ is the set of survivors, $\mathcal{N}$ is the set of existing plants, σ_I is the elasticity of substitution among surviving products, σ_N is the elasticity of substitution among new products, and σ_B is the elasticity of substitution between all the surviving and all the new products. In this case σ_B is the elasticity that should be used in our market share approach. With $\sigma_I = \sigma_N = \sigma_B$ we are back to the CES case in our core analysis. This we see as the most realistic case to the extent that there is no obvious reason to believe that surviving and new products should differ (surviving products are products that have been new at some point in the past too).

D.2 Varying markups

Our baseline analysis carries over to the case where markups are heterogeneous but uncorrelated with the age of the firm or with whether or not there was a successful innovation (own incumbent or new entrant innovation).

Now, suppose that: (i) the markups of unchanged products grow at gross rate g; (ii) the markups of new varieties are equal to g_n times the "average markup" in the economy in the last period; (iii) markups grow at gross rate g_i if there is an incumbent own innovation; (iv) markups after a successful creative destruction innovation is g_d times the markup of the eclipsed product. This amounts to replacing Assumption 1 in the main text by:[4]

$$\frac{q_{t+1}(j)}{\mu_{t+1}(j)} = \frac{\gamma_n}{g_n} \left(\frac{1}{N_t} \int_0^{N_t} \left(\frac{q_t(i)}{\mu_t(i)} \right)^{\sigma-1} di \right)^{\frac{1}{\sigma-1}}, \ \forall j \in (N_t, N_{t+1}].$$

Under the above assumptions the market share approach can still provide a precise estimate of missing growth, as long as: (a) we still make the assumption that the statistical office is measuring changes in markups of surviving product properly since changes in nominal prices are observed; (b) the market share relates to the quality-adjusted price in the same way for young and old firms,

[4]Note that this covers several possible theories governing the dynamics of markups. In particular it covers the case where firms face a competitive fringe from the producer at the next lower quality rung, in which $g_i > 1$ and $g < 1$. It also covers the case where newly born plants start with a low markup and markups just grow over the live-cycle of a product, in which $g_d < 1$, $g_n < 1$ and $g > 1$.

but recall that we are focusing our market share analysis on plants that have appeared in the data set for at least five years.

However, allowing for changing markups affects the expression for missing growth, which now becomes:

$$MG = \frac{1}{\sigma - 1} \log \left(1 + \frac{\lambda_d \left[\left(\frac{\gamma_d}{g_d}\right)^{\sigma-1} - g^{1-\sigma} - \lambda_i \left(\left(\frac{\gamma_i}{g_i}\right)^{\sigma-1} - g^{1-\sigma} \right) \right] + \lambda_n \left(\frac{\gamma_n}{g_n}\right)^{\sigma-1}}{g^{1-\sigma} + \lambda_i \left(\left(\frac{\gamma_i}{g_i}\right)^{\sigma-1} - g^{1-\sigma} \right)} \right).$$

In particular, allowing for changing markups introduces an additional source of missing growth having to do with the fact that the subsample of (surviving) products are not representative of all firms in their markup dynamics. For example, even if $\lambda_i = 1$ and $\gamma_i = \gamma_d$, there can be missing growth from creative destruction if the markup of creatively destroyed goods grows slower than the markup of products with incumbent own innovation, i.e., if $g_d < g_i$.

E Missing growth with capital

The purpose of this section of the Online Appendix is to extend our "missing growth" framework to a production technology with capital as an input, and to see how this affects estimated missing growth as a fraction of "true" growth.

E.1 A simple Cobb-Douglas technology with capital

Instead of the linear technology in the main text, we assume the following Cobb-Douglas production technology for intermediate inputs

$$y(j) = (k(j)/\alpha)^{\alpha} \left(l(j)/(1-\alpha)\right)^{1-\alpha}.$$

It is straightforward to see how this generalization affects the main equations in the paper. If R denotes the rental rate of capital, then the true aggregate price index becomes

$$P = p\left(\int_0^N q(j)^{\sigma-1}dj\right)^{\frac{1}{1-\sigma}},$$

with just $p = p(j) = \mu R^{\alpha} W^{1-\alpha}$.

Again we assume that the statistical office perfectly observes the nominal price growth $\frac{p_{t+1}(j)}{p_t(j)}$ of the surviving incumbent products. Since the Cobb-Douglas production technologies are identical across all intermediate inputs the capital-labor ratio equalizes across all firms and we have in equilibrium

$$y(j) = (\alpha)^{-\alpha}(1-\alpha)^{-(1-\alpha)}\left(\frac{K}{L}\right)^{\alpha} l(j),$$

where K and L denote the aggregate capital and labor stocks in the economy.

We assume that labor supply is constant over time and we assume a closed economy where profits, Π, labor earnings and capital income are spent on the final output good such that

$$P \cdot Y = W \cdot L + R \cdot K + \Pi.$$

Then we can derive the equilibrium output of an intermediate input j (the analog of expression (9) in the main text), which yields

$$y_t(j) = (\alpha)^{-\alpha}(1-\alpha)^{-(1-\alpha)} K_t^{\alpha} L^{1-\alpha} q_t(j)^{\sigma-1}\left(\int_0^{N_t} q_t(j'^{\sigma-1}dj'\right)^{-1}. \tag{17}$$

The aggregate production function can now be written in reduced form as

$$Y_t = (\alpha)^{-\alpha}(1-\alpha)^{-(1-\alpha)} Q_t K_t^{\alpha} L^{1-\alpha},$$

where $Q_t \equiv \left(\int_0^{N_t} q_t(j)^{\sigma-1}dj\right)^{\frac{1}{\sigma-1}}$. The term Q_t summarizes how quality/variety gains affect total productivity for given capital stock K_t.

Allowing for capital does not change anything in the model-based market share approach since we still have

$$\frac{S_{I_t,t+1}}{S_{I_t,t}} = \left(\frac{P_{t+1}}{P_t}\right)^{\sigma-1}\left(\widehat{\frac{P_{t+1}}{P_t}}\right)^{-(\sigma-1)}.$$

This equation can (still) be used to estimate missing growth as in Proposition 6 in the main text.[5] Hence the missing growth figures we obtained in Section 3.1.3 of the main text are unaffected when we introduce capital as specified above. The only important thing to note here is that this missing growth is "missing growth in the Q term" since under the assumption that nominal price growth is perfectly well observed by the statistical office we have:

$$MG = \left(\frac{P_t}{P_{t+1}}\right)\left(\widehat{\frac{P_{t+1}}{P_t}}\right) = \left(\frac{Q_{t+1}}{Q_t}\right)\left(\widehat{\frac{Q_t}{Q_{t+1}}}\right).$$

What may (potentially) change when introducing capital is how this missing growth should be compared to measured productivity growth. This issue is discussed in the remaining sections of this Online Appendix.

E.2 Finding "true" growth

So far we saw that our market share analysis in the main text remains valid when introducing capital, in the sense that it allows us to compute the bias in $\frac{Q_{t+1}}{Q_t}$. We now want to combine this missing growth estimate with information on measured growth to calculate "true" growth. The main question then is: what is the "right" estimate for measured growth $\left(\widehat{\frac{Q_{t+1}}{Q_t}}\right)$? Once we have found this "right" estimate of measured growth we can simply calculate true growth as

$$\left(\frac{Q_{t+1}}{Q_t}\right) = MG \cdot \left(\widehat{\frac{Q_{t+1}}{Q_t}}\right), \tag{18}$$

where MG is 1.0056 for the whole period in the baseline specification.

[5]This also easily generalizes to any constant return to scale production function.

A potentially difficulty here is that the capital stock, K_t, may itself grow over time.[6] Suppose K_t is growing at a constant rate over time, then part of the aggregate output growth $\frac{Y_{t+1}}{Y_t}$ is generated by capital deepening. Relatedly, if the capital stock grows over time the question arises as to whether this capital growth is perfectly measured or not. Finally, the long-run growth path of the capital stock will also matter and consequently we need to specify the saving and investment behaviors which underlie this growth of capital stock, and also need to take a stand as to whether there is investment specific technical change etc. The answer to all these questions have implication for the interpretation of the measured TFP growth and how it relates to $\widehat{\frac{Q_{t+1}}{Q_t}}$.

We first assume that the long-run growth rate of K_t results from a constant (exogenous) saving rate and abstract from investment specific technical change (see Section E.2.1). Furthermore we assume that all growth due to capital deepening is perfectly well observed and measured by the statistical office (see Section E.2.2) . Then, in Section E.2.3, we consider two alternative assumptions as to which part of physical capital growth is measured and analyze how these affect true growth estimates.

E.2.1 Capital accumulation

We assume that the final output good can be either consumed or invested. Furthermore we assume a constant exogenous saving/investment rate in the economy (we thus abstract from intertemporal optimization), i.e.,

$$K_{t+1} = K_t(1-\delta) + sY_t, \tag{19}$$

where s is the constant savings rate and δ is the depreciation rate of capital.

Suppose that $Q_{t+1}/Q_t = g$ is constant over time. This in turn implies that in

[6] If instead K_t was like "land", i.e., constant over time then the measured $\left(\widehat{\frac{Q_{t+1}}{Q_t}}\right)$ would be equal to the measured Hicks-neutral TFP growth.

the long run the capital-output ratio will stabilize at

$$\frac{K}{Y} = \frac{s}{g^{\frac{1}{1-\alpha}} - 1 + \delta}. \tag{20}$$

Along this balanced growth path investment, capital, and wages all grow at the same constant gross rate $g^{\frac{1}{1-\alpha}}$.

E.2.2 Measured output growth

Under the above assumption for capital accumulation, in the long run, true output growth is given by

$$\frac{Y_{t+1}}{Y_t} = \frac{Q_{t+1}}{Q_t}\left(\frac{Q_{t+1}}{Q_t}\right)^{\frac{\alpha}{1-\alpha}}. \tag{21}$$

Note that the first term on the right-hand side captures direct quality/variety gains, whereas the second term captures output growth due to capital deepening. In the following we assume that the second term is perfectly well measured whereas the first term is mismeasured as specified in our theory.[7] Under this assumption, measured output growth is equal to

$$\widehat{\frac{Y_{t+1}}{Y_t}} = \widehat{\frac{Q_{t+1}}{Q_t}}\left(\frac{Q_{t+1}}{Q_t}\right)^{\frac{\alpha}{1-\alpha}}. \tag{22}$$

E.2.3 Two alternative approaches on measured growth in capital stock

Next, we need to take a stand on how to measure the growth rate of capital stock. For given measured capital growth, the statistical office can compute the rate of

[7] This assumption rests on the view that the part of growth driven by capital deepening materializes—for given quality and variety—in increasing $y(j)$ (see (17)) which the statistical office should be able to capture (otherwise we would have still another source of missing growth).

Hicks-neutral TFP growth implicitly through the following equation:

$$\widehat{\frac{Q_{t+1}}{Q_t}}\left(\frac{Q_{t+1}}{Q_t}\right)^{\frac{\alpha}{1-\alpha}} = \left(\widehat{\frac{K_{t+1}}{K_t}}\right)^{\alpha}\widehat{\frac{TFP_{t+1}}{TFP_t}}. \tag{23}$$

First "macro" approach Here we assume that the bias in the measure of capital stock is the same as that for measuring real output.[8] Then the measured growth rate of capital stock in the long run is equal to

$$\widehat{\frac{K_{t+1}}{K_t}} = \widehat{\frac{Y_{t+1}}{Y_t}} = \widehat{\frac{Q_{t+1}}{Q_t}}\left(\frac{Q_{t+1}}{Q_t}\right)^{\frac{\alpha}{1-\alpha}}. \tag{24}$$

Substituting this expression for measured capital growth in (23) in turn yields

$$\widehat{\frac{TFP_{t+1}}{TFP_t}} = \left(\widehat{\frac{Q_{t+1}}{Q_t}}\right)^{1-\alpha}\left(\frac{Q_{t+1}}{Q_t}\right)^{\alpha}. \tag{25}$$

Substituting this into (18) then leads to:

$$\left(\frac{Q_{t+1}}{Q_t}\right)^{\frac{1}{1-\alpha}} = MG\cdot\left(\widehat{\frac{TFP_{t+1}}{TFP_t}}\right)^{\frac{1}{1-\alpha}}. \tag{26}$$

In other words, one should add MG to measured growth in TFP (in labor augmenting units) to get total "true" quality/variety growth in labor augmenting units. This is exactly what we are doing in our core analysis in the main text. Thus under the assumptions underlying this first approach the whole analysis and quantification of missing growth in our core analysis carries over to the extended model with capital. Let us repeat what underlies this approach: first, the focus is on the long-run when the capital-output ratio stabilizes at its balanced growth level; second, investment specific technical change is ruled out, so that the bias in measuring the growth in capital stock is

[8] This is a reasonable assumption to the extent that: (i) the same final good serves both as consumption good and as investment good; (ii) if the long-run growth rate of Q_t is constant, i.e., $Q_{t+1}/Q_t = g$, then the bias in measuring capital stock growth (when using a perpetual inventory method) is in the long run identical to the bias in measuring real output growth.

the same as that in measuring the growth in real output.[9]

Second "micro" approach Here we assume that the growth in capital stock is perfectly measured by the statistical office, i.e.,[10]

$$\frac{\widehat{K_{t+1}}}{K_t} = \left(\frac{Q_{t+1}}{Q_t}\right)^{\frac{1}{1-\alpha}}. \tag{28}$$

Plugging this expression in (23) gives

$$\frac{\widehat{TFP_{t+1}}}{TFP_t} = \frac{\widehat{Q_{t+1}}}{Q_t}, \tag{29}$$

so that

$$\frac{Q_{t+1}}{Q_t} = MG \cdot \frac{\widehat{TFP_{t+1}}}{TFP_t}. \tag{30}$$

This in turn implies that our missing growth estimate should be added to measured TFP growth in Hick-neutral terms to obtain Hicks-neutral "true" TFP growth. Assuming $\alpha = 1/3$, this approach would increase missing growth as a fraction of true growth from 25% ($= 0.64/(1.87 + 0.64)$) see Table 1 in the main text) to 34% ($= 0.64/(1.87 \cdot 2/3 + 0.64)$).

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[9]To get some intuition, note that we can also write the production function as

$$Y_t = (\alpha)^{-\alpha} (1-\alpha)^{-(1-\alpha)} Q_t^{\frac{1}{1-\alpha}} \left(\frac{K_t}{Y_t}\right)^{\frac{\alpha}{1-\alpha}} L. \tag{27}$$

Since under the assumptions above the growth rate in the capital-output ratio, $\frac{K_t}{Y_t}$, (which is zero in the long run) is properly measured, we see that missing growth automatically obtains a labor-augmenting interpretation and should consequently be compared to TFP growth estimates expressed in labor augmenting terms.

[10]We see this approach as being more "micro" for the following reason. Suppose we only have data about the only one industry. Then we could use our market share approach together with data about the revenue shares of different products to estimate missing output growth in this particular industry. It would then be reasonable to compare this number to the Hicks-neutral TFP growth in this industry, within the implicit assumption that the statistical office perfectly measures the growth in capital stock in the industry when calculating TFP growth. Next, one could sum-up "missing growth" and measured Hicks-neutral TFP growth to compute "true" TFP growth. This true TFP growth would of course itself be mismeasured if there is mismeasurement in the growth of capital stock: this would add yet another source of missing growth.

E.3 Wrapping-up

In this Appendix we argued that our core analysis can easily be extended to production technologies involving physical capital. Under our first (macro) approach the missing growth estimates remain exactly the same as in our core analysis based on the model without capital. And moving to our second (micro) approach only increases our missing growth estimates. In that sense, the macro approach can be viewed as being more conservative.

F Other robustness checks

The gains from variety Our theory does not impose much discipline in terms of how the gains from specialization/variety are calibrated. Our baseline specification makes the standard assumption connecting the gains from specialization to the elasticity of substitution. It assumes the increasing the available product variety by one percent increases final output by $1/(\sigma - 1)$ percent. This only affects missing growth from variety expansion. In our second quantification approach, in the next section, we show that missing growth mainly originates from creative destruction as opposed to variety expansion. Consequently, we expect this assumption not to be as critical as it first seems.

Bias in measuring incumbent own innovation Proposition 6 assumes that quality improvements from incumbent own innovation are correctly measured, i.e., that $\widehat{\gamma}_i = \gamma_i$ and $\widehat{\lambda}_i = \lambda_i$. Without this assumption, missing growth in our model is given by

$$MG_{t+1} = \frac{1}{\sigma - 1} \left[\log \left(\frac{1 + \lambda_i(\gamma_i^{\sigma-1} - 1)}{1 + \widehat{\lambda}_i(\widehat{\gamma}_i^{\sigma-1} - 1)} \right) + \log \left(\frac{S_{I_t,t}}{S_{I_t,t+1}} \right) \right]. \quad (31)$$

Understating incumbent own innovation adds log-linearly to missing growth, contributing directly and making the bias from imputation larger.

Imports and outsourcing Our model did not taken into account the possibility that plants may outsource the production of some items to other plants. Nor did it consider the role of imports as an additional source of new products. On outsourcing, our answer is twofold: (i) if the outsourcing is to another incumbent plant or leads an incumbent plant to shut down, then then outsourcing will not affect our analysis and results; (ii) if outsourcing is to a new plant then it can be viewed as an instance of creative destruction since the reason for such outsourcing is presumably that the new plant produces at lower (quality-adjusted) price; it will be treated as such in our market share approach.

Outsourcing may indeed create a bias in our missing growth estimates if incumbent plants survive but outsource overseas. Our LBD dataset only covers domestic employment.[11]

Finally, imports are known to affect manufacturing the most, as manufacturing goods are the most tradable. Very little of our missing growth, however, comes from manufacturing (see Table 5). This suggests that overall missing growth is not affected much by what happens in import-competing sectors.

G Manufacturing vs. non-manufacturing

In the paper, we reported missing growth by the market share method for all sectors in the economy. We also calculated missing growth within manufacturing and non-manufacturing sectors. Table 1 displays the result. In

[11] Domestic M&A should not affect missing growth in the same way because we are looking at plants, not firms. If firm A acquires firm B and all firm B plants remain in operation, then these plants will be counted as surviving plants. If some of firm B's plants close as a result of the M&A, then we rightly count them as exiting. One might want to compute the fraction of aggregate missing growth associated with M&A, but we leave that for future research.

the first column, we reiterate the baseline results in the market share section of our paper. The second and third columns report missing growth in manufacturing and non-manufacturing, respectively. Missing growth in non-manufacturing is about 0.11 percentage points larger than our baseline results but also appears to be constant over time. Missing growth in manufacturing, however, is only 0.04 percentage points on average between 1983–2013.

Table 1: MANUFACTURING VS. NON-MANUFACTURING

	All	Mfg	Non-mfg
1983–2013	**0.64**	**0.04**	**0.75**
1983–1995	0.66	0.15	0.81
1996–2005	0.55	-0.04	0.65
2006–2013	0.74	-0.03	0.79

Notes: This table presents missing growth estimates for the whole 1983–2013 period (as well as different sub-periods) by manufacturing and non-manufacturing sectors. The growth numbers are expressed in (average) percentage points per year. The results in column "All" are identical to the baseline results in the paper. The elasticity of substitution, σ, is 4 and the lag, k, is 5 throughout the table. Manufacturing are NAICS 31–33 plants. Non-manufacturing are all other sectors excluding farming NAICS 01 and public sector NAICS 09.

H Implementation of GHK

H.1 Our notation vs. GHK code notation

Table 2: GHK NOTATIONS VS. OUR NOTATION

Parameter	Our model	GHK equivalent
Share of non-obsolete products with OI innovation	$\lambda_i(1-\lambda_d)$	$\frac{\lambda_i}{(1-\delta_o)}$
Share of non-obsolete products having incumbent CD	0	$\frac{\delta_i(1-\lambda_i)}{(1-\delta_o)}$
Share of non-obsolete products having entrant CD	λ_d	$\frac{\delta_e(1-\lambda_i)}{(1-\delta_o)}$
Measure of incumbent or entrant NV in $t+1$ relative to the number of products in t	λ_n	$\kappa_i+\kappa_e+\delta_o$
Share of obsolescence	0	δ_o
Net expected step size of CD innovation	$\gamma_d^{\sigma-1}-1$	$\frac{1-\delta_o}{1-\delta_o\psi}(E[s_q^{\sigma-1}]-1)$
Net expected step size of OI innovation	$\gamma_i^{\sigma-1}-1$	$\frac{1-\delta_o}{1-\delta_o\psi}(E[s_q^{\sigma-1}]-1)$
Quality of NV innovation relative to average productivity last period	γ_n	$s\kappa^{\frac{1}{\sigma-1}}$
Average quality of product becoming obsolete in $t+1$ relative to average quality in t	n/a	ψ
Elasticity of substitution	σ	σ

H.2 Changes to the GHK algorithm

We made the following changes to the GHK algorithm.

1. The original GHK methodology assumes that the statistical office measures growth perfectly. Hence, the algorithm chooses parameters such that true growth, given by equation (14), matches measured growth in the data. We modify the algorithm to allow measured and true growth to differ. Instead of matching to true growth, we choose parameters so that measured growth in (15) matches the observed growth rates: 1.66% for 1983–1993, 2.29% for 1993–2003 and 1.32% for 2003–2013.

2. We impose an additional restriction that comes from the CPI micro data. We restrict the sum of the (unconditional) arrival rates of OI and CD to equal the cumulative rate of non-comparable substitutions from the CPI over 5 years. This substitution rate averages 3.75% per 2 months in the CPI.[12] Using the notation in our market share model, we impose that $\lambda_i(1-\lambda_{e,d}-\lambda_{i,d})+\lambda_{i,d}+\lambda_{e,d}=0.68$.[13]

3. Since the original GHK code estimates 5-year arrival rates and step sizes, whereas BLS substitutions and imputations happen at a monthly or bimonthly frequency (depending on the item), we convert 5-year arrival rates into bimonthly arrival rates by imposing $(1-X^{(b)})^{30}=1-X^{(5)}$, where $X^{(b)}$ and $X^{(5)}$ denote the bimonthly and five-year arrival rates, respectively. We then scale the bimonthly OI and CD arrival rates in equal proportion so that their sum equals the bimonthly CPI non-comparable substitution rate of 3.75%. Finally, we adjust the step sizes of NV and CD so that: (i) annualized bimonthly measured growth equals the observed annual measured growth; and (ii) the relative contributions of CD and NV to growth stay the same as those estimated using 5-year parameters.

[12] Klenow and Kryvtsov (2008).
[13] $0.68=1-(1-0.0375)^{30}$. 30 compounds the bi-monthly arrival rate to 60 months (5 years).

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